Remarks on quantum entanglement

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Abstract

Some remarks on the concept of quantum entanglement are presented. A more physical and natural notion is proposed. It is equivalent to an algebraic definition which is close to the non decomposability of overall states into tensor products. The paradigm of quantum entanglement of mutually distant particles, which arose from the EPR paradox, is critically considered. It is shown that the results of Aspect's experiments, possibly, were misinterpreted.

§ 1. Amended notion of quantum entanglement.

Consider a numbered set of n quantum particles. Particle number s can be in one of the eigenstates $|x^s_{j_s}\rangle$ of a complete set of commuting observables, where $j_s \in \{1, \dots, N_s\}$ and $s \in \{1, ..., n\}$. If one considers this ensemble as a single quantum object, then its state may be represented as $|x^1_{j_1}\rangle \dots |x^n_{j_n}\rangle = |x^1_{j_1} \dots x^n_{j_n}\rangle$. This implies that any tensor

$$|A\rangle = \sum_{j_1,\dots,j_n} c_{j_1\dots j_n} \cdot |x^1_{j_1}\rangle \otimes \dots \otimes |x^n_{j_n}\rangle = \sum_{j_1,\dots,j_n} c_{j_1\dots j_n} \cdot |x^1_{j_1} \dots x^n_{j_n}\rangle \tag{1}$$

represents a state of the particle collection. It is assumed that $\langle x^n_{i_n} \dots x^1_{i_1} | x^1_{j_1} \dots x^n_{j_n}\rangle = \delta_{i_1 j_1} \dots \delta_{i_n j_n}$. Then the state $|A\rangle$ is normalized iff $\sum_J |c_{j_1\dots j_n}|^2 = 1$. The condition of symmetry or antisymmetry should be imposed on the tensor (1) in the case of identical bosons or fermions correspondingly. The set of indices $j_1, \dots, j_{k-1}, j_{k+1}, \dots, j_n$ we denote as $j_1, \dots, \widehat{j_k}, \dots, j_n$.

But there are no physical grounds for the opinion that *any* collection of quantum systems with the state spaces S_j is an object with the state space $S = \bigotimes_j S_j$. This preconception, being a basis of the quantum computing paradigm, leads to strange conclusions. For example, the pair of electrons selected from different galaxies is treated as a united quantum system. Apparently, the tensor products of state spaces should be introduced more cautiously.

Definition 1 *Let a normalized vector (1) represents an overall quantum state of particles with numbers $1, 2, \dots, n$. Then particles k and l, where $l > k$, are called entangled in the state $|A\rangle$ if there exists such a pair i_k, i_l that the following holds.*

In the case $n > 2$

$$\sum_{j_1,\ldots,\widehat{j}_k,\ldots,\widehat{j}_l,\ldots,j_n} |c_{j_1\ldots i_k\ldots i_l\ldots j_n}|^2 \neq \sum_{j_1,\ldots,\widehat{j}_k,\ldots,j_n} |c_{j_1\ldots i_k\ldots j_l\ldots j_n}|^2 \cdot \sum_{j_1,\ldots,\widehat{j}_l,\ldots,j_n} |c_{j_1\ldots j_k\ldots i_l\ldots j_n}|^2 \ , \tag{2}$$

in the case $n = 2$ (when $k = 1$ and $l = 2$)

$$|c_{i_1 i_2}|^2 \neq \sum_{j_2} |c_{i_1 j_2}|^2 \cdot \sum_{j_1} |c_{j_1 i_2}|^2 \ .$$

A pair of not entangled particles is called independent (in the corresponding state (1)). A set of particles is called independent if each pair of them is independent.

Let as a result of some measurement the particles k and l have been transferred into states $|x^k_{i_k}\rangle$ and $|x^l_{i_l}\rangle$. Denote these random events as A_k and B_l. Then the left-hand and right-hand parts (2) are equal to $P(A_k B_l)$ and $P(A_k)P(B_l)$ accordingly. Equation (2) means that A_k and B_l are mutually dependent events (in the sense of the theory of probabilities). The independence of all such events is equivalent to the independence of particles k and l in the state (1). This clarifies the physical meaning of Definition 1.

Proposition 1 *A set of two particles in a state $|A\rangle = \sum_{j_1 j_2} c_{j_1 j_2} |x_{j_1} y_{j_2}\rangle$ is independent iff for some $v_{j_1}, w_{j_2} \in \mathbb{C}$ and $\varphi_{j_1,j_2} \in \mathbb{R}$ the following holds:*

$$\sum_{j_1 j_2} c_{j_1 j_2} e^{i\varphi_{j_1 j_2}} \cdot |x_{j_1} y_{j_2}\rangle = \sum_{j_1} v_{j_1} |x_{j_1}\rangle \otimes \sum_{j_2} w_{j_2} |y_{j_2}\rangle \ . \tag{3}$$

Proof. We assume the state vector $|A\rangle$ to be normalized.

Suppose (3) takes place. Then $c_{j_1 j_2} e^{i\varphi_{j_1 j_2}} = v_{j_1} w_{j_2}$ and for all k, l we have:

$$|c_{kl}|^2 = |c_{kl}|^2 \cdot \sum_{j_1 j_2} |c_{j_1 j_2}|^2 = |v_k|^2 |w_l|^2 \cdot \sum_{j_2} |w_{j_2}|^2 \cdot \sum_{j_1} |v_{j_1}|^2 = \sum_{j_2} |c_{k j_2}|^2 \cdot \sum_{j_1} |c_{j_1 l}|^2 \ .$$

By Definition 1 the particles are independent.

Conversely, let $|c_{kl}|^2 = \sum_{j_2} |c_{k j_2}|^2 \cdot \sum_{j_1} |c_{j_1 l}|^2$ for all k, l. Then denote $|v_k|^2 = \sum_{j_2} |c_{k j_2}|^2$ and $|w_l|^2 = \sum_{j_1} |c_{j_1 l}|^2$. The complex numbers v_k and w_l are defined up to phase factors. Arbitrarily fixating these factors we have $|c_{kl}| = |v_k w_l|$ and, hence, each c_{kl} differs from $v_k w_l$ by a multiplier $e^{i\varphi_{kl}}$ $\square$.

Definition 2 *Let a set of particles in a state (1) is the union of two non-empty and disjoint subsets A and B. These subsets are called entangled (with each other) if there exists a pair of entangled particles $a \in A$ and $b \in B$. Otherwise they are called independent. If a particle set cannot be separated into independent subsets, then it is called entangled.*

The existence of independent subsets means that the particle set is a formal union of quantum systems not interacting with each other. This physically evident fact strictly follows from the following assertion.

Theorem 1 *Let a set of n particles in a state (1) is separated into two non-empty subsets. The particles are numbered so that $1, 2, \ldots, m$ constitute one subset and $m+1, m+2, \ldots, n$ form the other.*

These subsets are independent in the state (1) iff for some $v_{j_1 \ldots j_m}$, $w_{j_{m+1} \ldots j_n} \in \mathbb{C}$ and $\widetilde{c}_{j_1 \ldots j_n} = c_{j_1 \ldots j_n} e^{i\varphi_{j_1, \ldots, j_n}}$ with $\varphi_{j_1, \ldots, j_n} \in \mathbb{R}$ the following holds:

$$\sum_{j_1, \ldots, j_n} \widetilde{c}_{j_1 \ldots j_n} |x^1_{j_1} \ldots x^n_{j_n}\rangle = \sum_{j_1, \ldots, j_m} v_{j_1 \ldots j_m} |x^1_{j_1} \ldots x^m_{j_m}\rangle \otimes \sum_{j_{m+1}, \ldots, j_n} w_{j_{m+1} \ldots j_n} |x^{m+1}_{j_{m+1}} \ldots x^n_{j_n}\rangle. \tag{4}$$

Proof. We assume the state vector $|A\rangle$ to be normalized.

Suppose (4) takes place. Let's verify the independence of any two particles with numbers $k \leq m$ and $l \geq m+1$. From (4) it follows that

$$\sum_{j_1, \ldots, \widehat{j}_k, \ldots, \widehat{j}_l, \ldots, j_n} |c_{j_1 \ldots i_k \ldots i_l \ldots j_n}|^2 = \sum_{j_1, \ldots, \widehat{j}_k, \ldots, j_m} |v_{j_1 \ldots i_k \ldots j_m}|^2 \cdot \sum_{j_{m+1}, \ldots, \widehat{j}_l, \ldots, j_n} |w_{j_{m+1} \ldots i_l \ldots j_n}|^2. \tag{5}$$

The right-hand part (2) can be calculated as follows:

$$\sum_{j_1, \ldots, \widehat{j}_k, \ldots, j_n} |c_{j_1 \ldots i_k \ldots j_l \ldots j_n}|^2 \cdot \sum_{j_1, \ldots, \widehat{j}_l, \ldots, j_n} |c_{j_1 \ldots j_k \ldots i_l \ldots j_n}|^2 =$$

$$\sum_{j_1, \ldots, \widehat{j}_k, \ldots, j_m} |v_{j_1 \ldots i_k \ldots j_m}|^2 \cdot \sum_{j_{m+1}, \ldots, j_n} |w_{j_{m+1} \ldots j_n}|^2 \cdot \sum_{j_1, \ldots, j_m} |v_{j_1 \ldots j_m}|^2 \cdot \sum_{j_{m+1}, \ldots, \widehat{j}_l, \ldots, j_n} |w_{j_{m+1} \ldots i_l \ldots j_n}|^2 =$$

$$\sum_{j_1, \ldots, \widehat{j}_k, \ldots, j_m} |v_{j_1 \ldots i_k \ldots j_m}|^2 \cdot \sum_{j_{m+1}, \ldots, \widehat{j}_l, \ldots, j_n} |w_{j_{m+1} \ldots i_l \ldots j_n}|^2 \cdot \sum_{j_1, \ldots, j_n} |c_{j_1 \ldots j_n}|^2.$$

As the latter factor is equal to 1, we have obtained the right-hand part of (5). According to Definition 1 the particles k and l are independent in state (1).

Suppose now that two particles with numbers $k \leq m$ and $l \geq m+1$ are independent in this state. Renumbering the set, if necessary, one may assume that this pair stands at the beginning. Then the state vector looks as follows:

$$|A\rangle = \sum_{j_3, \ldots, j_n} \Big(\sum_{j_1, j_2} c_{j_1 j_2 j_3 \ldots j_n} \cdot |x^1_{j_1} x^2_{j_2}\rangle\Big) \otimes |x^3_{j_3} \ldots x^n_{j_n}\rangle$$

For any fixed indices $j_3 \ldots j_n$ let's consider the particles 1 and 2 as a set in the quantum state $\sum_{j_1, j_2} c_{j_1 j_2 j_3 \ldots j_n} \cdot |x^1_{j_1} x^2_{j_2}\rangle$. This set of particles is independent (Definition 1). Proposition 1 implies that $c_{j_1 j_2 j_3 \ldots j_n} = \widetilde{v}_{j_1 j_3 \ldots j_n} \cdot \widetilde{w}_{j_2 j_3 \ldots j_n}$ for all j_1, j_2. Returning back to the initial numeration of all the particles, the overall state is represented by the tensor

$$|A\rangle = \sum_{j_1, \ldots, j_n} v_{j_1 \ldots \widehat{j_l} \ldots j_n} \cdot w_{j_1 \ldots \widehat{j_k} \ldots j_n} \cdot |x^1_{j_1} \ldots x^n_{j_n}\rangle. \tag{6}$$

If there exists another pair of particles from the independent subsets with numbers $r \leq m$ and $s \geq m+1$, then by the analogy to (6) we obtain $c_{j_1 \ldots j_n} = a_{j_1 \ldots \widehat{j_s} \ldots j_n} \cdot b_{j_1 \ldots \widehat{j_r} \ldots j_n}$. As $v_{j_1 \ldots \widehat{j_l} \ldots j_n} \cdot w_{j_1 \ldots \widehat{j_k} \ldots j_n} = a_{j_1 \ldots \widehat{j_s} \ldots j_n} \cdot b_{j_1 \ldots \widehat{j_r} \ldots j_n}$, then for some $\lambda \neq 0$ we have:

$$\begin{aligned} v_{j_1 \ldots \widehat{j_l} \ldots j_n} &= \lambda(j_1, \ldots, \widehat{j_l}, \ldots, j_n) \cdot a_{j_1 \ldots \widehat{j_s} \ldots j_n} \\ w_{j_1 \ldots \widehat{j_k} \ldots j_n} &= \lambda^{-1}(j_1, \ldots, \widehat{j_k}, \ldots, j_n) \cdot b_{j_1 \ldots \widehat{j_r} \ldots j_n} \end{aligned}$$

It is seen that a and b don't depend on j_l and j_k correspondingly. Thus

$$|A\rangle = \sum_{j_1, \ldots, j_n} a_{j_1 \ldots \widehat{j_l} \ldots \widehat{j_s} \ldots j_n} \cdot b_{j_1 \ldots \widehat{j_k} \ldots \widehat{j_r} \ldots j_n} \cdot |x^1_{j_1} \ldots x^n_{j_n}\rangle.$$

The mutual order of the particles k and r as well as l and s does not matter. This process will stop once one of the independent subsets has become empty, i.e., all its particles have been chosen before. Obviously, equation (4) will be obtained in such a way $\Box$.

Corollary 1 *set of n particles is independent in a state (1) iff for some $v_1^{j_1}, \ldots, v_n^{j_n} \in \mathbb{C}$ and $\widetilde{c}_{j_1 \ldots j_n} = c_{j_1 \ldots j_n} e^{i\varphi_{j_1, \ldots, j_n}}$ with $\varphi_{j_1, \ldots, j_n} \in \mathbb{R}$ the following holds:*

$$\sum_{j_1, \ldots, j_n} \widetilde{c}_{j_1 \ldots j_n} |x^1_{j_1} \ldots x^n_{j_n}\rangle = \sum_{j_1} v_1^{j_1} |x^1_{j_1}\rangle \otimes \ldots \otimes \sum_{j_n} v_n^{j_n} |x^n_{j_n}\rangle.$$

Proof. The independence of a set is equivalent to the pairwise independence of all the particles it contains. As easy to verify, an independent pair inside a set will be independent in any subset containing the pair. The proof residue directly follows from Theorem 1 $\Box$.

Proposition 2 *The fact that a particle set is entangled in some state implies that this quantum system cannot be separated into not interacting subsystems.*

Speaking of not interacting systems we mean the absence of any physical interaction, neither directly nor by means of other bodies. For a triple of particles the criterion of independence looks as follows:

$$\begin{aligned} \textstyle\sum_j |c_{jkl}|^2 &= \textstyle\sum_{jr} |c_{jkr}|^2 \cdot \sum_{js} |c_{jsl}|^2 \\ \textstyle\sum_j |c_{kjl}|^2 &= \textstyle\sum_{jr} |c_{kjr}|^2 \cdot \sum_{js} |c_{sjl}|^2 \\ \textstyle\sum_j |c_{klj}|^2 &= \textstyle\sum_{jr} |c_{krj}|^2 \cdot \sum_{js} |c_{slj}|^2 \end{aligned} \tag{7}$$

Violation at least one of (7) indicates that the corresponding particles k and l are entangled in this state. In the case of qubits equations (7) can be easily verified by a computer.

Definition 1 for the entangled state of two particles is close to the conventional notion. Namely, a pair state $|A\rangle$ is called separable or entangled depending on whether it is decomposable into the tensor product (3) or not. Proposition 1 implies that any pair of separable particles is independent. The converse is not true as seen from the following example. Consider a pair state

$$|A_\varphi\rangle = |0\rangle \otimes |0\rangle + e^{i\varphi} \cdot |0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle \tag{8}$$

with $\varphi \in [0; 2\pi)$. The state $|A_0\rangle = \big(|0\rangle + |1\rangle\big) \otimes \big(|0\rangle + |1\rangle\big)$ is separable. According to Proposition 1, these particles are independent in any state (8). But the latter is not separable if $\varphi \neq 0$.

It is important to note that for any φ_1 and φ_2 the states $|A_{\varphi_1}\rangle$ and $|A_{\varphi_2}\rangle$ (8) are experimentally indistinguishable. Indeed, according to the foundations of quantum mechanics [1], in the course of any measurement of this pair the state $|A_\varphi\rangle$ will equiprobably be collapsed into either of the states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$ regardless of the value φ. Analogously, let there be given a state $|A\rangle = \sum_{kl} c_{kl} \cdot |x_k y_l\rangle$ of any two particles. The difference from a state with the coefficients c_{kl} multiplied by any phase factors is not detectable in practice. For example, let $|x\rangle$ and $|y\rangle$ be the linear polarization states of a photon. Then the circular polarization state $(|x\rangle + i|y\rangle)/\sqrt{2}$ and the state $(|x\rangle + |y\rangle)/\sqrt{2}$ of equiprobable polarizations along the axes x and y can not be distinguished by means of any polarizer. Indeed, in both these states a photon will pass through the polarizer with equal probabilities.

Thus, the usual definition of entangled state as one that is not separable [2] results in the situation when physically independent particles are considered to be entangled. The emended notion of quantum entanglement is a few stronger. According to Definitions 1 and 2, if mutually distant particles are considered to be entangled, then correlations between the results of some their measurements must a'priori be detectable in experiments. Herewith the usual paradigm predicts such correlations as a logical consequence of a purely mathematical concept based on the tensor product notion.

In what follows a precise definition of quantum entanglement is not significant and a reader may keep in mind the concept which is habitual for him.

§ 2. EPR – entanglement

In what follows an entangled state of mutually distant particles which don't physically interact to each other is called EPR – entangled. This idea goes back to the EPR paradox, and the present article is questioning the possibility of such a state. But here we are considering EPR – entangled states as theoretically possible ones.

This paradigm results in conclusions that are so contrary to the common sense that the popular term of quantum magic looks justified. It makes the picture of the world very spectacular. In addition, the notion of EPR – entanglement is critically important to quantum computing. This concept is a basis for the control of qubits and the organization of quantum parallelism. According to this paradigm, EPR – entanglement and quantum entanglement are the same concepts.

The most of experiments on the confirmation of EPR – entanglement phenomenon are related to the interference of photons. In this regard, the following quote of Dirac is worth to be noted (§3 Chapter 1 [1]).

"*...Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the intensity of a beam is connected with the probable number of photons in it, we should have half the total number of photons going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components. Each photon then interferes only with itself. Interference between two different photons never occurs.*"

Thus, all the attempts of detecting EPR – entangled photons by using interferometers are meaningless. Indeed, each photon can interfere only with itself. Hence, the detection of coincident polarizations may not be interpreted as an evidence of EPR – entanglement. Hence, one should critically analyze all the experiments relating to entangled photons [3]. The same relates to the experiments with photons entangled in phases [4], because a photon has no observables with the eigenvalues of the phase.

A pair of electrons, that have jumped out of an atomic orbital in the course of ionization, is often considered to be an example of EPR – entangled particles. These electrons are leaving the atom in an overall state which is entangled in spins, i.e., in the state $\big(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle\big)/\sqrt{2}$ where the signs $+$ and $-$ indicate the spin projections onto z - axis. It is believed that such a state should arise by virtue of the law of angular momentum conservation.

Suppose the atom has had zero total angular momentum before ionization. One may assume that the electrons are flying apart being in the independent state

$$|A\rangle = \frac{(\alpha_1|+\rangle + \beta_1|-\rangle) \otimes (\alpha_2|+\rangle + \beta_2|-\rangle)}{\sqrt{|\alpha_1|^2|\alpha_2|^2 + |\alpha_1|^2|\beta_2|^2 + |\alpha_2|^2|\beta_1|^2 + |\beta_1|^2|\beta_2|^2}} \ .$$

Then the average value of the overall angular momentum is equal to

$$\langle A|s_1 \otimes I_2 + I_1 \otimes s_2|A\rangle = \frac{(|\alpha_1|^2|\alpha_2|^2 - |\beta_1|^2|\beta_2|^2)\,\hbar}{\sqrt{|\alpha_1|^2|\alpha_2|^2 + |\alpha_1|^2|\beta_2|^2 + |\alpha_2|^2|\beta_1|^2 + |\beta_1|^2|\beta_2|^2}} \tag{9}$$

where s_1, s_2 are the spin operators and I_1, I_2 are the identity operators acting in the state spaces of electrons taken separately. Here the angular momentum conservation means that (9) is zero, i.e., $|\alpha_1||\alpha_2| = |\beta_1||\beta_2|$. With $|\alpha_1|^2 + |\beta_1|^2 = 1$ and $|\alpha_2|^2 + |\beta_2|^2 = 1$ from here it follows that $\forall r \geq 0$ one may assume the following:

$$|\alpha_1| = \frac{r}{\sqrt{1+r^2}}\ , \quad |\beta_1| = \frac{1}{\sqrt{1+r^2}}\ , \quad |\alpha_2| = \frac{1}{\sqrt{1+r^2}}\ , \quad |\beta_2| = \frac{r}{\sqrt{1+r^2}}\ .$$

Thus, there exists continuum of independent states of the pair that do not contradict the law of angular momentum conservation. Hence, this pair of electrons is by no means obliged to be entangled. The converse opinion, in fact, is a common preconception.

It is strange that violations of Bell's inequalities are considered to be the evidences in favor of the EPR – entanglement phenomenon. Such violations do occur but, as seen from the fountainhead [5], this allows making only one of two the following conclusions.

a) Quantum systems have no hidden parameters [6]. This meets to the "orthodoxal" quantum mechanics [1] and is not related to EPR – entanglement.

b) Hidden parameters exist and a measurement of one particle may affect the other far remote particle. Hence, EPR – entanglement might have a place in reality.

One may assume that violations of Bell's inequalities entail the assertion a). However, these violations are ubiquitously considered to be the evidences of b). This point of view was formed under the influence of experiments by Aspect and similar ones. Allegedly, in such experiments the correlations between the linear polarizations of mutually remote photons were being observed [7]. Be this is true, Bell's inequalities would not be needed for experimental verification of the EPR – entanglement phenomenon.

§ 3. Aspect's experiments.

The data obtained in these experiments are usually interpreted on the basis of the interpretation of photons as corpuscles, i.e., well localized particles. Fluorescent light sources were used, where an atom emits two photons with the average interval $\tau \approx 5$ ns. It is believed that the photons from each atom have the same circular polarizations and are flying apart in opposite directions. According to this classic picture, the angular momentum of the pair is zero. From here Aspect [7] concluded that its state is entangled in linear polarizations:

$$|A\rangle = \frac{|x\rangle \otimes |x\rangle + |y\rangle \otimes |y\rangle}{\sqrt{2}} = \frac{|R_1\rangle \otimes |R_2\rangle + |L_1\rangle \otimes |L_2\rangle}{\sqrt{2}} \tag{10}$$

where the photon 1 is moving in the direction of z - axis whilst the photon 2 is moving backwards, $|R_1\rangle = |L_2\rangle = \big(|x\rangle + i|y\rangle\big)/\sqrt{2}$ and $|L_1\rangle = |R_2\rangle = \big(|x\rangle - i|y\rangle\big)/\sqrt{2}$. The states $|x\rangle$ and $|y\rangle$ of a single photon correspond to its linear polarizations along x and y axes. In the states $|L_j\rangle$ and $|R_j\rangle$ the photon j is circularly polarized clockwise and counterclockwise correspondingly, if one looks from the side of z - axis when $j = 1$ and from the opposite side when $j = 2$.

Equality (10) is proved by trivial algebraic calculations but a non-trivial mistake is hiding here. Namely, any eigenstate of polarization is meaningful only in some eigenstate of momentum $\mathbf{p}$. Then the photon wave function looks as

$$\mathbf{f}(\mathbf{k}, t) = \frac{i}{\sqrt{\Delta}} \cdot \delta_{\mathbf{kp}} e^{-i\omega t} \cdot \mathbf{e} \tag{11}$$

with a polarization vector $\mathbf{e} \in \mathbb{C}^3$, where $(\mathbf{e}, \mathbf{p}) = 0$ and $(\mathbf{e}^*, \mathbf{e}) = 1$ (2.4) [8]. Here Δ is the volume of a sufficiently large cube containing all the electromagnetic field, $\mathbf{p} = \omega_0 \mathbf{n}$ with $\mathbf{n} \in \mathbb{Z}^3$, $|\mathbf{n}| \gg 1$ and $\omega_0 = 2\pi/\Delta^{1/3}$ (the Heaviside system of units is combined with the relativistic one, so that $\hbar = c = 1$ [8]).

In the case of electric dipole radiation, the emitted photon is assumed to be in some eigenstate of angular momentum. According to [7], the corresponding quantum numbers are $j = 1$ and $M = \pm 1$ for the pair of photons emitted in one cascade. The eigenstates of angular momentum with a frequency ω are represented by the wave functions

$$\mathbf{f}(\mathbf{k}, t) = \frac{ik}{\omega\sqrt{\delta j(j+1)}} \cdot \delta_{k\omega} e^{-i\omega t} \cdot \nabla_{\mathbf{k}} Y_{jM} \tag{12}$$

where Y_{jM} is a spherical harmonic with $j > 0$ (4,15)(4.20) [8]. Here $\delta = \pi/R$ and R is the radius of a large ball containing all the electromagnetic field, $\omega = \omega_0 n$ with $n \in \mathbb{N}$, $n \gg 1$ and $\omega_0 = \delta$.

But a photon can not simultaneously be in the eigenstates of momentum and angular momentum because these observables don't commute. Easy to see that wave functions (11) and (12) are totally different. Thus, in [7] there was expounded a theoretically incorrect argumentation in favor of entangled pairs appearance. The classic model, where two emitted photons with the same circular polarizations are flying apart in opposite directions, in fact is far from QED.

It is worth to mention that analogous simplification is met in chapter 18 of the legendary book [9]. It might be acceptable for the first acquaintance to quantum mechanics, but such kind of reasonings play an important part in the interpretations of experiments on EPR – entanglement.

The experiment by Aspect reduces to counting photon pairs passed through two mutually distant polarizers [7]. A short time interval ~ 5 ns between two detections in a row is considered to be the guarantee that both these photons were emitted by a single atom. But one may interpret this experiment in a different way. It is reasonable to suppose that two detector operations in a row are related to a single photon in the form of a "spherical wave" which interacts with *two* photomultiplier tubes. Let's consider such a picture in details.

In the eigensate of angular momentum and its z - projection, the electric component of the photon field according to (12) looks as follows:

$$\mathbf{E}(\mathbf{r},t) = 2\Re\Big\{i\sqrt{\frac{\delta}{2}}\Big(\frac{\omega}{2\pi}\Big)^{3/2}\cdot\int_{|\mathbf{n}|=1}\mathbf{Y}^{(1)}_{jM}(\mathbf{n})e^{i\omega(\mathbf{n}\cdot\mathbf{r}-t)}dS(\mathbf{n})\Big\} \tag{13}$$

where $\mathbf{Y}^{(1)}_{jM} = k(j(j+1))^{-1/2}\cdot\nabla_{\mathbf{k}}Y_{jM}$ (4.15) [8]. Since $j=1$ in the case under consideration, then in view of (4.18) [8] $\mathbf{Y}^{(1)}_{jM}(-\mathbf{n}) = \mathbf{Y}^{(1)}_{jM}(\mathbf{n})$ and from (13) we have $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(-\mathbf{r},t)$. Hence, this photon will equally interact with two equally oriented polarizers at the points $\pm\mathbf{r}$.

In reality these polarizers are differently removed from the light source. Let the distances are $r_2 > r_1$ and $\tau = r_2 - r_1$ (here $c=1$). Suppose the field of a photon represents a spherical wave and $(a;b)$ is a time interval. Then $\forall t\in(a;b)$ we have $kr_2 - \omega(t+\tau) = kr_1 - \omega t$ and $\mathbf{E}(\mathbf{r}_1,t) = \mathbf{E}(\mathbf{r}_2,t+\tau)$. Hence, this photon is equally interacting with both the polarizers.

But the photon field (13) is not a spherical wave, although it propagates in all directions from the source. In the case $j=1$, the following asymptotic formula is obtained from (4.28) [8] for the field (13) in the "wave zone":

$$\mathbf{E}(\mathbf{r},t)\approx 8\pi\Re\Big\{i\sqrt{\frac{\delta}{2}}\Big(\frac{\omega}{2\pi}\Big)^{3/2}\cdot\Big(\sqrt{1/3}\cdot\mathbf{Y}_{1,2,M}+\sqrt{2/3}\cdot\mathbf{Y}_{1,0,M}\Big)\frac{\sin(kr)}{kr}\cdot e^{-i\omega t}\Big\} \tag{14}$$

where $\mathbf{Y}_{1,l,M}$ is a vector spherical harmonic with $M=\pm1$. According to (14), the field $\mathbf{E}$ has two components with phases $kr-\omega t$, $kr+\omega t$ and equal amplitudes. The component with the phase $kr-\omega t$ interacts with the polarizers just like a spherical wave.

As for the other component, the phases difference $(kr_2+\omega(t+\tau)) - (kr_1+\omega t) = 2\omega\tau$ $\forall t\in(a;b)$. The values r_1 and r_2 are defined approximately and, obviously, the error $\Delta\tau = \Delta(r_2-r_1)\gg\lambda$ where λ is the photon wavelength. Then $\Delta(2\omega\tau)\gg 1$ and, when considering the interaction with a polarizer, the component with the phase $kr+\omega t$ is similar to a photon with zero degree of polarization and, hence, may be ignored.

Thus, considering a photon with quantum numbers ω, $j=1$ and $M=\pm1$ as the wave (13), one must conclude that it will interact with both the polarizers similarly to a pair of equally polarized photons flying apart towards these polarizers (in opposite directions). From the point of view of QM the following happens. After passing through the nearest polarizer, the photon found itself in the state of a definite polarization but indefinite direction of the motion. This picture will not seem strange if one takes into account that the observables of polarization and momentum represent some linear operators commuting with each other [1]. Then the photon freely had passed through the second polarizer which has the same orientation.

Let's come back to the ordinary system of units. In Aspect's experiments the wavelengths of the photons were close to 420 and 550 nm which corresponds to the energies ≈ 3 and 2.3 eV. Since the sensibility of photocathodes achieves ≈ 1.1 eV, the energy of one such photon is sufficient for activating two photomultipliers. This event is by no means exclusive because there exist photocathodes with the quantum yield of 2 electrons per a photon with $\lambda \approx 1.4$ μm.

The Einstein's equation allows a photon to punch out only one electron. This corresponds to the case $A < \hbar\omega < 2A$, where A is the work function. Evidently, this inequality was true in early experiments on photoeffect. Indeed, for a metallic surface $A > 3$ eV and only some alkaline and alkaline earth metals have $A \approx 2 - 3$ eV. In classical experiments by Stoletow the ultraviolet light $\lambda \approx 295$ nm, which corresponds to $\hbar\omega \approx 4.2$ eV, was used [10].

The principles of QM [1], in fact, does not forbid a photon to interact with several electrons. Such effects were observed in much more dramatic situations with super-power laser beams in the soft X-ray and extreme ultraviolet range [11, 12]. Quite reasonable to suppose that, when a photon in state (13) meets two photomultipliers at two places remote from each other, it spends only a part of the energy in every interaction as if there have been two acts of inelastic (Compton) scattering.

According to (56.12) [13], for hydrogen-like atoms the photoeffect cross section

$$\sigma = \frac{2^9\pi^2}{3}\frac{\alpha a_0^2}{Z^2}\left(\frac{I}{\hbar\omega}\right)^4 \frac{e^{-4\nu\arctan(1/\nu)}}{1 - e^{-2\pi\nu}} \qquad \nu = \frac{Ze^2}{\hbar v} = \frac{Z\alpha}{v/c}\,, \quad \alpha \approx \frac{1}{137} \tag{15}$$

where v is the electron velocity after departing from an atom. Under the conditions of Aspect's experiment a photoelectron velocity does not exceed ~ 1000 km/sec and, as easy to verify, the multiplier in (15) containing ν parameter may be considered to be constant. Hence, the cross-section σ does not depend on the frequency ω provided $\hbar\omega > A$. One may suppose that the first interaction of photon — photomultiplier, albeit has reduced ω, has not affected the cross section of the second interaction.

But why is the photon counter activated two times in a row with the average interval ≈ 5 ns ? For detecting of the photon pairs two photomultipliers were used [7]. The rise time of the photoelectron avalanche in this device is easy to estimate as ~ 10 ns. Only one photon can be detected within this period. In fact the photon represents not a wave (13) but a packet of such waves. If the wave packet size $\Delta r \sim 1$ meter, which corresponds to the Doppler broadening $\sim 10^{-3}$ Å, then the time of passing through the photomultiplier has the same order of magnitude as the interval between two photons emitted from an atom. The wave packet, propagating in all directions from the source, passes through both the polarizers and then activates both the photomultipliers. Until they finish their operations, i.e., during ~ 10 ns no other photons can be detected. When the photomultipliers restored the operability, the wave packet of the second photon has already passed by.

Thus, quite reasonable looks the supposition that in most the cases not entangled pairs but single photons were detected by means of the pair of photomultipliers. Thus, the results of Aspects' experiments can be explained without introducing the concept of EPR – entanglement. All the other experiments with allegedly EPR – entangled photons, presumably, can be interpreted in analogous way.

§ 4. Conclusion

The EPR – entanglement, i.e., the widespread concept of quantum entanglement is now considered as a scientific fact which was theoretically deduced from quantum mechanics and reliably confirmed by experiments. Most of the scientific community have accepted this idea as is, despite of the fact that many of the predicted effects defy the common sense and deserve the epithet of quantum magic. The aim of the article was to show that this paradigm has weak theoretical and experimental grounds. Quantum entanglement is really the place to be in systems of identical particles which overall states satisfy the condition of symmetry or antisymmetry [1]. But one should distinguish such systems, arising in a natural way, with those defined by means of arbitrary and formal associations of quantum objects.

The emended notion of quantum entanglement, that was introduced in §1, demands quantum correlations to really take place when they are considered. This definition is close to the usual one that introduces the quantum entanglement by means of the notion of tensor product. But Definition 1 looks more physical and, possibly, it would reduce the freedom of purely mathematical speculations around the quantum entanglement.

The classical experiments by Aspect became an experimental foundation for the EPR – entanglement paradigm (§2). As it is shown in §3, they can be interpreted without using this concept. Furthermore, there are no reliable, theoretical grounds for the conventional interpretation of these experiments in the terms of entangled photon pairs. Since the allegedly entangled photons are considered to be the main evidence in favor of the EPR — entanglement phenomenon, one should critically reconsider such experiments and more cautiously make use of this notion.

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